15 September 1997

ELSEVIER

Optics Communications 141 (1997) 237–242

OPTICS COMMUNICATIONS

Reciprocity theorem for Smith-Purcell radiation

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Received 30 January 1997; accepted 13 May 1997

Abstract

Smith-Purcell radiation is produced when a charged particle moves close and parallel to a diffraction grating. Calculation of Smith-Purcell spectra is therefore linked to a special grating problem, involving incident evanescent waves describing the field of the moving electron. For large period gratings, radiation in the far-infrared and millimetric range is produced, in a spectral range where metallic gratings can often be considered as infinitely conducting. In that case, conservation laws and a reciprocity theorem are derived for Smith-Purcell radiation by applying techniques from electromagnetic grating theory. © 1997 Elsevier Science B.V.

PACS: 41.60.–m; 42.25.Fx; 42.79.Dj
Keywords: Diffraction; Smith-Purcell effect; Grating

1. Introduction

In 1953 Smith and Purcell [1] observed light emission by a fast electron beam passing close to a periodic structure. Several theories have been proposed to explain the properties of the observed radiation. Oscillating charges or dipoles [1–3] have been considered, as well as periodic current sheets of deflected electrons [4]. In 1961, Toraldo di Francia [5] applied the concept of diffraction of evanescent waves to explain Smith-Purcell (SP) radiation from shallow gratings, but at that time no rigorous theory was available for arbitrary grating profiles. His model was later on improved by van den Berg [6] who applied a full electromagnetic diffraction theory [7,8] to the special case of the SP effect. According to Gover, Dvorkis and Elisha [9] who made a comparison of different models describing the Smith-Purcell effect, the van den Berg model fits best the experimental data, as long as the electron beam does not hit the grating, producing other types of radiation [10–14]. Recent experiments with higher energies electrons [15–17] reattracted the interest on SP radiation as a potential source of far infrared radiation and the theoretical description of the phenomenon has been extended by considering a charged particle moving parallel to the grating surface at an arbitrary angle with respect to the grating rulings [18]. A spectacular property of perfectly conducting gratings in spectroscopy is described by the so-called reciprocity theorem [19]: the efficiency in the zero order of diffraction does not change when the grating is rotated by 180° around an axis perpendicular to its surface. Together with the energy conservation theorem, the reciprocity theorem shows that as long as there are only two propagative diffracted orders (i.e. the zero order at specular reflexion and the -1 order), the efficiency in the -1 order is also insensitive to a 180° rotation of the grating. The purpose of this work is to establish the reciprocity theorem for Smith-Purcell radiation.

2. Smith-Purcell theory

Fig. 1 gives a schematical description of a Smith-Purcell experiment at arbitrary incidence angle to the rulings and of the relevant geometrical quantities. The electron moves

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PII S0030-4018(97)00269-1

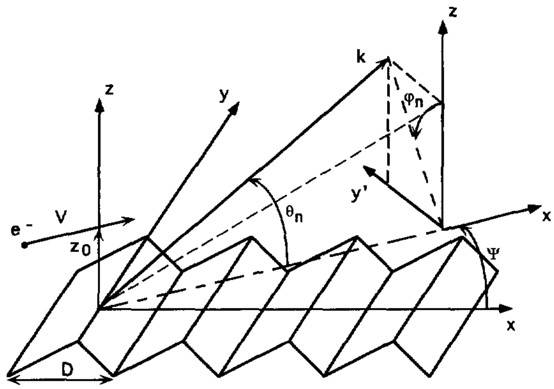

Fig. 1. Schematical layout of the geometry of a Smith-Purcell experiment. The electron moves parallel to a perfecly conducting grating, with constant speed. The trajectory of the electron makes an angle Ψ with the grating rulings.

in vacuum parallel to the surface of a perfectly conducting grating. The grating profile is described by a periodic function $z=f(x)=f(x+D)$ with the direction of the rulings parallel to the y axis and the top of the grating lies in the (x,y) plane. The electron moves along a trajectory $z=z_0=$ const and Ψ is the angle between the x axis and the projection of the electron trajectory onto the $(x$–$y)$ plane. The electron moves with constant velocity $\mathbf{v}=v(\cos\Psi\ \mathbf{i}_x+\sin\Psi\ \mathbf{i}_y)$. $\mathbf{i}_x,\mathbf{i}_y,\mathbf{i}_z$ are unit vectors in the x,y,z direction, respectively. We adopt the conventions used in Refs. [6,18,21], to which the reader is referred for further details. The incident real field vectors $\mathbf{E}^{\mathrm{i}}(x,y,z,t)$ and $\mathbf{H}^{\mathrm{i}}(x,y,z,t)$ are expanded in terms of Fourier integrals:

$$\mathbf{E}^{\mathrm{i}}(x,y,z,t)=(2\pi^2)^{-1}\mathrm{Re}\left[\int_0^{+\infty}\mathrm{d}\omega\int_0^{\infty}\mathrm{d}\beta\,\boldsymbol{E}^{\mathrm{i}}(x,z;\beta,\omega)\times\exp(\mathrm{i}\beta y-\mathrm{i}\omega t)\right], \quad (1)$$

$$\mathbf{H}^{\mathrm{i}}(x,y,z,t)=(2\pi^2)^{-1}\mathrm{Re}\left[\int_0^{+\infty}\mathrm{d}\omega\int_0^{\infty}\mathrm{d}\beta\,\boldsymbol{H}^{\mathrm{i}}(x,z;\beta,\omega)\times\exp(\mathrm{i}\beta y-\mathrm{i}\omega t)\right]. \quad (2)$$

The Fourier components satisfy the Maxwell equations which take the following form:

$$(\nabla+\mathrm{i}\beta\mathbf{i}_y)\times\boldsymbol{H}^{\mathrm{i}}+\mathrm{i}\omega\epsilon_0\boldsymbol{E}^{\mathrm{i}}=\boldsymbol{J}, \quad (3)$$

$$(\nabla+\mathrm{i}\beta\mathbf{i}_y)\times\boldsymbol{E}^{\mathrm{i}}-\mathrm{i}\omega\mu_0\boldsymbol{H}^{\mathrm{i}}=0, \quad (4)$$

with $\nabla=\partial_x\mathbf{i}_x+\partial_z\mathbf{i}_z$ and $\boldsymbol{J}=\boldsymbol{J}(x,z;\beta,\omega)$ being the Fourier transform of the current density distribution:

$$\boldsymbol{J}(x,z;\beta,\omega)=qv\,\delta(z-z_0)\{\mathbf{i}_x+\mathbf{i}_y\tan\Psi\}\times\exp\left[\mathrm{i}(\omega-\beta v\sin\Psi)\frac{x}{v\cos\Psi}\right]. \quad (5)$$

The Maxwell equations show that the x and z components of the Fourier field vectors $\boldsymbol{E}^{\mathrm{i}}$ and $\boldsymbol{H}^{\mathrm{i}}$ can be expressed as functions of their y components which satisfy the two-dimensional Helmholtz equations:

$$(\partial_x^2+\partial_z^2)E_y^{\mathrm{i}}+(k^2-\beta^2)E_y^{\mathrm{i}}=-\mathrm{i}\omega\mu_0J_y+\frac{\beta}{\omega\epsilon_0}(\partial_xJ_x+\mathrm{i}\beta J_y), \quad (6)$$

$$(\partial_x^2+\partial_z^2)H_y^{\mathrm{i}}+(k^2-\beta^2)H_y^{\mathrm{i}}=-\partial_zJ_x. \quad (7)$$

Here $k=\omega/c$ (c is the speed of lightin vacuum). We obtain the solutions of these equations as:

$$H_y^{\mathrm{i}}(x,z;\beta,\omega)=-\frac{q}{2}\mathrm{sign}(z-z_0)\exp(\mathrm{i}\alpha_0x+\mathrm{i}\gamma_0|z-z_0|), \quad (8)$$

$$E_y^{\mathrm{i}}(x,z;\beta,\omega)=\frac{q}{2}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2}\left(\frac{\beta c/v-k\sin\Psi}{\gamma_0\cos\Psi}\right)\times\exp(\mathrm{i}\alpha_0x+\mathrm{i}\gamma_0|z-z_0|), \quad (9)$$

in which $\alpha_0=\omega/(v\cos\Psi)-\beta\tan\Psi$ and $\gamma_0=\mathrm{i}(\alpha_0^2+\beta^2-k^2)^{1/2}$ with $(\alpha_0^2+\beta^2-k^2)^{1/2}>0$. As a consequence of the limited speed of the electron $v<c$, $\alpha_0^2+\beta^2>k^2$. Therefore, γ_0 is always imaginary and nonzero, which means that the electromagnetic field generated by the moving electron is represented by a set of evanescent plane waves exponentially decaying in the direction away from the electron trajectory. The reflected fields $\mathbf{E}^{\mathrm{r}}$ and $\mathbf{H}^{\mathrm{r}}$ are also expanded as Fourier integrals in which the Fourier transforms $\boldsymbol{E}^{\mathrm{r}}$, $\boldsymbol{H}^{\mathrm{r}}$ satisfy the source-free Maxwell equations:

$$(\nabla+\mathrm{i}\beta\mathbf{i}_y)\times\boldsymbol{E}^{\mathrm{i}}-\mathrm{i}\omega\mu_0\boldsymbol{H}^{\mathrm{i}}=0, \quad (10)$$

$$(\nabla+\mathrm{i}\beta\mathbf{i}_y)\times\boldsymbol{H}^{\mathrm{i}}+\mathrm{i}\omega\epsilon_0\boldsymbol{E}^{\mathrm{i}}=0, \quad (11)$$

and the boundary condition for a perfectly conducting surface $\mathbf{n}\times(\boldsymbol{E}^{\mathrm{i}}+\boldsymbol{E}^{\mathrm{r}})=0$, in which $\mathbf{n}$ is the unit vector normal to the surface (see Fig. 1). The three-dimensional vectorial problem is separated into two scalar problems of two dimensions called the two fundamental cases of polarization, viz. the E-polarization and the H-polarization. The y components of the electric and magnetic fields are expressed in terms of Rayleigh expansions [20]:

$$E_y^{\mathrm{r}}(x,z;\beta,\omega)=\sum_{n=-\infty}^{+\infty}E_{y,n}^{\mathrm{r}}(\beta,\omega)\exp(\mathrm{i}\alpha_nx+\mathrm{i}\gamma_nz), \quad (12)$$

$$H_y^{\mathrm{r}}(x,z;\beta,\omega)=\sum_{n=-\infty}^{+\infty}H_{y,n}^{\mathrm{r}}(\beta,\omega)\exp(\mathrm{i}\alpha_nx+\mathrm{i}\gamma_nz), \quad (13)$$

in which $\alpha_n=\alpha_0+2\pi n/D$ and $\gamma_n=(k^2-\beta^2-\alpha_n^2)^{1/2}$

with $\mathrm{Re}(\gamma_n) \geq 0$ and $\mathrm{Im}(\gamma_n) \geq 0$. In an experiment, the detector is usually at a large distance from the grating so that neither the evanescent field of the electron hitting directly the detector nor the evanescent components of the Rayleigh expansions are detected. Then, the Smith-Purcell radiation is made up of the diffracted propagating waves, i.e. those waves for which $\mathrm{Im}(\gamma_n) = 0$. Therefore, one must have $\alpha_n^2 + \beta^2 \leq k^2$ and β is restricted to $|\beta| \leq k$. As a consequence, $\alpha_0 > k$ and only some of the negative orders $n < 0$ from the Rayleigh expansions are propagative and contribute to the SP spectrum. Generally one chooses for grating problems angles of incidence and emergence which are linked to the grating axes. For SP diffraction problems however, it is more natural to choose emergence angles linked to the electron trajectory. Considering waves with fixed parameter k_0 and introducing angles of emergence (θ, φ) linked to the electron trajectory (see Fig. 1), one has for propagating waves:

$$\alpha_n = k(\cos\Psi \cos\theta_n - \sin\Psi \sin\theta_n \sin\varphi_n), \tag{14}$$

$$\beta = k(\sin\Psi \cos\theta_n + \cos\Psi \sin\theta_n \sin\varphi_n), \tag{15}$$

$$\gamma_n = k \sin\theta_n \cos\varphi_n, \tag{16}$$

and one obtains the Smith-Purcell dispersion relation:

$$-n\lambda = \frac{D}{\cos\Psi}\left(\frac{c}{v} - \cos\theta_n\right). \tag{17}$$

Radiation of constant wavelength is emitted along a cone of aperture θ_n centered on the projection of the electron trajectory onto the $(x$–$y)$ plane. For $\Psi = 0°$ and relativistic electrons ($E \gg E_0 = 0.511$ MeV), $v/c \approx 1$ and to tune the radiation, one has to change the angle of observation θ or to use a set of gratings with different periods. Rotating the grating changes the apparent grating period seen by the electron according to Eq. (17) and could constitute a more practical way of tuning the SP emission, as it would not require to change the set-up used to collect the radiation.

The real part of the complex Poynting vector represents the power density (power per unit area) radiated in spectral order n:

$$\tfrac{1}{2}\boldsymbol{E}_n^{\mathrm{r}} \times \boldsymbol{H}_n^{\mathrm{r}*} = \frac{1}{2}\frac{\omega}{k^2 - \beta^2}\left(\epsilon_0 |E_{y,n}^{\mathrm{r}}|^2 + \mu_0 |H_{y,n}^{\mathrm{r}}|^2\right)\mathbf{k}_n, \tag{18}$$

in which $\mathbf{k}_n = (\alpha_n, \beta, \gamma_n)$ is the wave vector of the diffracted radiating wave of order n. Introducing the radiation factor (analogous to the reflection coefficient in spectroscopy) and the interaction range [8,18]

$$|R_n(\theta,\varphi,\Psi)|^2 = \frac{4}{e^2(1-\Delta^2)}\left\{\epsilon_0/\mu_0 |E_{y,n}^{\mathrm{r}}|^2 + |H_{y,n}^{\mathrm{r}}|^2\right\} \times \exp(2|\gamma_0| z_0), \tag{19}$$

$$h_{\mathrm{int}}^n(\theta,\varphi,\Psi) = \frac{1}{2|\gamma_0|} = \frac{\lambda_n \cos\Psi}{4\pi}\left[(c/v - \Delta\sin\Psi)^2 + \cos^2\Psi(\Delta^2 - 1)\right]^{-1/2}, \tag{20}$$

with

$$\Delta = \beta/k = \sin\Psi \cos\theta + \cos\Psi \sin\theta \sin\varphi, \tag{21}$$

one can derive the power emitted in a given spectral order n into a solid angle $\mathrm{d}\Omega$ in direction (θ,φ) by the electron passing by one grating period [2]:

$$\left(\frac{\mathrm{d}P}{\mathrm{d}\Omega}\right)_n = \frac{q^2 D^2}{2\epsilon_0 |n| \lambda^3} |R_n(\theta,\varphi,\Psi)|^2 \sin^2\theta \cos^2\varphi \times \exp(-z_0/h_{\mathrm{int}}^n). \tag{22}$$

The parameter h_{int} can be considered as an effective interaction range: if the electron passes within the interaction range, it effectively contributes to the nth order Smith-Purcell radiation in the (θ,φ) direction. If it passes far from the grating (i.e. several times the interaction range) the electron will not produce Smith-Purcell radiation with a significant intensity. The z_0 dependence of the incident field is introduced explicitly in the definition of the radiation factor. With this definition of the radiation factor, the total power of SP radiation emitted by an electron beam is obtained integrating (22) analytically over the beam profile [9,18,21] and multiplying by the total number of grooves of the grating. When a pulsed beam is used and the wavelength of the emitted photons is comparable to the bunch length, coherence effects appear which greatly enhance the emitted power density [16,17].

3. Power relations and reciprocity theorem

We consider now the following two Smith-Purcell configurations of Figs. 2a, 2b. The electron moves with a speed $\mathbf{v} = v \cos\Psi\ \mathbf{i}_x + v \sin\Psi\ \mathbf{i}_y$ in the first case, and with a speed $\mathbf{v}' = -\mathbf{v}$ in the second case. All the other parameters are the same. The total fields above the grating $E_y = E_y^{\mathrm{i}} + E_y^{\mathrm{r}}$ and $H_y = H_y^{\mathrm{i}} + H_y^{\mathrm{r}}$ are given by Eqs. (8), (9) and (12), (13) in the first case. In the second case, the total fields above the grating when $0 \leq z < z_0$ are given by:

$$\mathscr{E}_y = -\frac{q}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{-\beta' c/v - k\sin\Psi}{\gamma_0' \cos\Psi}\right) \times \exp(\mathrm{i}\alpha_0' x - \mathrm{i}\gamma_0'(z - z_0)) + \sum_{m=-\infty}^{\infty} \mathscr{E}_{y,m}^{\mathrm{r}} \exp(\mathrm{i}\alpha_m' x + \mathrm{i}\gamma_m' z), \tag{23}$$

[2] In the corresponding formula for $\Psi = 0°$ published in Refs. [9,21] a factor $n^2/2$ is missing.

$$\mathscr{H}_y = -\frac{q}{2}\exp(\mathrm{i}\alpha_0' - \mathrm{i}\gamma_0'(z-z_0)) + \sum_{m=-\infty}^{\infty} \mathscr{H}^{\mathrm{r}}_{y,m}\exp(\mathrm{i}\alpha_m' x + \mathrm{i}\gamma_m' z), \qquad (24)$$

with $\alpha_0' = -\omega/(v\cos\Psi) - \beta'\tan\Psi$, $\gamma_0' = \mathrm{i}(\alpha_0'^2 + \beta'^2 - k^2)^{1/2}$, $\alpha_m' = \alpha_0' + 2\pi m/D$ and $\gamma_m' = (k^2 - \beta'^2 - \alpha_m'^2)^{1/2}$ with $\mathrm{Re}(\gamma_m') \geq 0$ and $\mathrm{Im}(\gamma_m') \geq 0$. With this definition, only some of the positive orders of diffraction $m > 0$ contribute to the Smith-Purcell radiation in the second configuration. To establish the power relations [6,18], one makes use of the following lemma: if S is a periodic perfectly conducting surface with period D described by $z = f(x)$ and $\{U, U'\}$ are two functions satisfying the Helmholtz equation and the Dirichlet boundary condition $\{U = 0, U' = 0\}$ or the Neumann boundary condition $\{\mathbf{n}\cdot\nabla U = 0,\ \mathbf{n}\cdot\nabla U' = 0\}$, then above the grating ($z \geq 0$) [19]:

$$\int_{x_1}^{x_1+D} \{U\partial_z U' - U'\partial_z U\}\,\mathrm{d}x = 0. \qquad (25)$$

We apply relation (25) to the couples of functions $\{E_y, E_y^*\}$, $\{H_y, H_y^*\}$, $\{\mathscr{E}_y, \mathscr{E}_y^*\}$ and $\{\mathscr{H}_y, \mathscr{H}_y^*\}$ (A^* being the complex conjugate of A). Noting that for a propagative order n, $\gamma_n^* = \gamma_n$ and for a nonpropagative order n, $\gamma_n^* = -\gamma_n$, one obtains:

$$\sum_{\gamma_n^2 \geq 0} |E^{\mathrm{r}}_{y,n}|^2 \gamma_n = q\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{\beta c/v - k\sin\Psi}{\gamma_0\cos\Psi}\right) \times \mathrm{Re}\big(E^{\mathrm{r}}_{y,0}\exp(\mathrm{i}\gamma_0 z_0)\big), \qquad (26)$$

$$\sum_{\gamma_n^2 \geq 0} |H^{\mathrm{r}}_{y,n}|^2 \gamma_n = -q\,\mathrm{Re}\big(H^{\mathrm{r}}_{y,0}\gamma_0\exp(\mathrm{i}\gamma_0 z_0)\big), \qquad (27)$$

$$\sum_{\gamma_m'^2 \geq 0} |\mathscr{E}^{\mathrm{r}}_{y,m}|^2 \gamma_m' = -q\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{-\beta' c/v - k\sin\Psi}{\gamma_0'\cos\Psi}\right) \times \mathrm{Re}\big(\mathscr{E}^{\mathrm{r}}_{y,0}\exp(\mathrm{i}\gamma_0' z_0)\big), \qquad (28)$$

$$\sum_{\gamma_m'^2 \geq 0} |\mathscr{H}^{\mathrm{r}}_{y,m}|^2 \gamma_m' = -q\,\mathrm{Re}\big(\mathscr{H}^{\mathrm{r}}_{y,0}\gamma_0'\exp(\mathrm{i}\gamma_0' z_0)\big). \qquad (29)$$

The summations are taken over propagative orders n or m only ($\gamma_n^2 \geq 0$ and $\gamma_m'^2 \geq 0$, respectively). In the "classical" grating problem, a grating is illuminated by an incident plane wave, and one calculates the distribution of the diffracted waves and their intensities. A first test to check the validity of the results is to apply the energy conservation theorem: the sum of the energies of the diffracted propagative orders has to be equal to the energy of the incident wave. In the special case of the Smith-Purcell diffraction this form of the energy conservation theorem does not hold because the incident waves are evanescent and carry no energy (the energy conservation theorem applies for the system electron plus electromagnetic field). Relations (26) and (27) are equivalent to the energy balance criterion [19] in spectroscopy and can be used to test the validity of the results. We will refer to them as the power relations for Smith-Purcell radiation.

Applying relation (25) to $\{\mathscr{E}_y(-\beta,\omega); E_y(\beta,\omega)\}$ and $\{\mathscr{H}_y(-\beta,\omega); E_y(\beta,\omega)\}$, one obtains:

$$\mathscr{E}^{\mathrm{r}}_{y,0}(-\beta,\omega) = -E^{\mathrm{r}}_{y,0}(\beta,\omega), \qquad (30)$$

$$\mathscr{H}^{\mathrm{r}}_{y,0}(-\beta,\omega) = -H^{\mathrm{r}}_{y,0}(\beta,\omega). \qquad (31)$$

These relations are equivalent to the reciprocity theorem applied to the zero order of diffraction in spectroscopy [19]. The difference is that in the SP case the zero order is evanescent, while it is propagative when using the grating for spectroscopy. An interesting consequence is obtained by considering a wavelength λ for which there is only one propagative diffracted order. The combination of the power relations (26)–(29) and the reciprocity relations (30) and (31) for the zero order gives the reciprocity relations for the first order of diffraction:

$$|\mathscr{E}^{\mathrm{r}}_{y,1}(-\beta,\omega)|^2 = |E^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2, \qquad (32)$$

$$|\mathscr{H}^{\mathrm{r}}_{y,1}(-\beta,\omega)|^2 = |H^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2. \qquad (33)$$

These relations mean that when there is only one propagative diffracted wave, the amplitude of this single wave is the same in the two configurations. In fact, Fig. 2c shows that ones passes from one configuration to the other by applying a 180° rotation around the z axis. Taking into account the expressions of the fields in both cases, one can finally obtain the following properties:

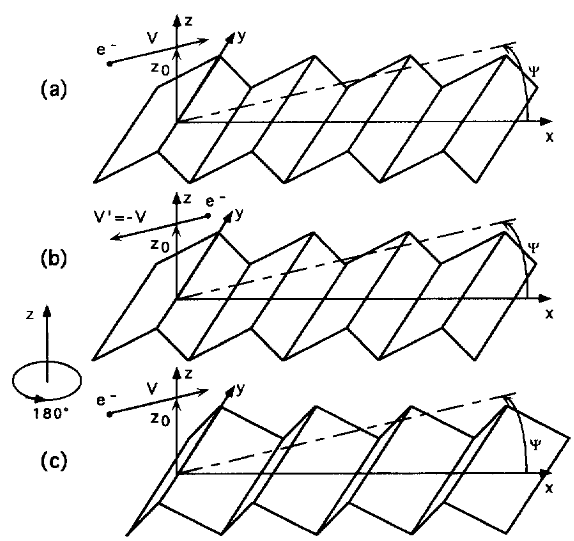

Fig. 2. Smith-Purcell configurations for the reciprocity theorem: (a) the electron moves at speed **v** parallel to the grating surface; (b) the electron moves at speed $-\mathbf{v}$ parallel to the grating surface; (c) a 180° rotation around the z axis applied to the first configuration leads to the second case and demonstrates the reciprocity theorem.

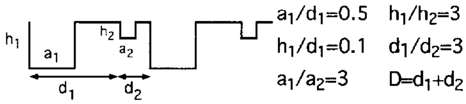

Fig. 3. Nonsymmetric grating profile used in the computations to illustrate the reciprocity theorem.

The coefficients of the zero order in the Rayleigh expansions remain the same when the grating is rotated by 180° around the z axis [22]:

$$E^{\mathrm{r}}_{y,0}(\beta,\omega) = {}_{180}E^{\mathrm{r}}_{y,0}(\beta,\omega), \tag{34}$$

$$H^{\mathrm{r}}_{y,0}(\beta,\omega) = {}_{180}H^{\mathrm{r}}_{y,0}(\beta,\omega). \tag{35}$$

If only the -1 order contributes to the Smith-Purcell radiation, its Rayleigh coefficients are the same after a 180° rotation of the grating:

$$|E^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2 = |{}_{180}E^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2, \tag{36}$$

$$|H^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2 = |{}_{180}H^{\mathrm{r}}_{y,-1}(\beta,\omega)|^2, \tag{37}$$

and therefore the Smith-Purcell radiation factor is insensitive to a 180° rotation of the grating:

$$|R_{-1}(\theta,\varphi)|^2 = |{}_{180}R_{-1}(\theta,\varphi)|^2. \tag{38}$$

In that case, both the polarization of SP radiation as well as the power density emitted by the electron remain the same. To the author's knowledge, these properties have never been observed experimentally.

We illustrate the invariance theorem for the following configuration: a 100 keV electron is interacting at an angle $\Psi = 10°$ with the nonsymmetric grating described in Fig. 3. For this type of profile, a modal analysis [23,24] is well adapted to solve the grating problem. Fig. 4 shows the results for the radiation factor $|R_{-1}(\theta,\varphi)|^2$ for $0 \leq \theta \leq 180°$ and for a fixed angle $\varphi = 20°$. Solid line is for the first case. Dashed line is for the second case after a 180° rotation of the grating. For $\theta > \theta_{-2} = 60.72°$, only one order is propagative and the curves are identical. At θ_{-2} a second order appears and a strong anomaly is visible on the curve. This type of anomaly belongs to the Wood-Rayleigh anomalies [25,26]. For lower angles, several propagative diffracted orders contribute to the Smith-Purcell spectrum and the curves are different.

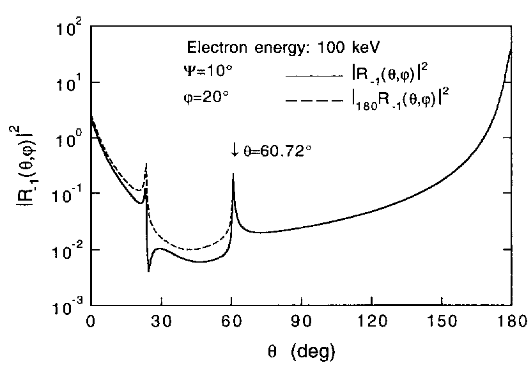

Fig. 4. Radiation factor $|R_{-1}(\theta,\varphi)|^2$ in the first order of diffraction at $\varphi = 20°$ when a 100 keV electron passes at an angle $\Psi = 10°$ over the grating of Fig. 3. Solid line is for $|R_{-1}(\theta,\varphi)|^2$, dashed line is for ${}_{180}|R_{-1}(\theta,\varphi)|^2$.

4. Conclusion

Solving a grating problem usually involves numerical approximations which can lead to convergence problems. In order to check the validity of the results, one makes use of the energy conservation theorem when convergence has been achieved. For nonsymmetric grating profiles, the reciprocity theorem is another test. For Smith-Purcell radiation the incident electromagnetic field is evanescent and usual expressions of these theorems cannot be used. Applying techniques from the electromagnetic theory of gratings, the analogue of the energy conservation theorem and of the reciprocity theorem in spectroscopy have been derived for the Smith-Purcell effect and can be used as a test for the accuracy of the computations. To the author's knowledge the predicted reciprocity properties have never been observed experimentally. In particular, in Smith-Purcell experiments with low energy electrons and optical gratings for emission in the near-infrared or visible spectral range [1,3,4,9–14], it seems difficult to avoid that electrons hit the grating and other radiation mechanisms like bremsstrahlung or transition radiation can contribute to the observed radiation [10–14]. Under these conditions, the reciprocity relations could also constitute an experimental test for the Smith-Purcell model developed by Van den Berg [6] and recently generalized by Haeberlé et al. [18].

Acknowledgements

The author would like to gratefully acknowledge Y. Takakura for valuable discussions on grating theory which led to this work, as well as N. Maene and P. Rullhusen for their support.

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